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THE DEPENDENCE OF HALO CLUSTERING ON HALO FORMATION HISTORY, CONCENTRATION, AND OCCUPATION

Risa H. Wechsler[1,2], Andrew R. Zentner[1], James S. Bullock[3], Andrey V. Kravtsov[1] Brandon Allgood[4]

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ABSTRACT

We investigate the dependence of dark matter halo clustering on halo formation time, density profile concentration, and subhalo occupation number, using high-resolution numerical simulations of a LCDM cosmology. We confirm results that halo clustering is a function of halo formation time at fixed mass, and that this trend depends on halo mass. For the first time, we show unequivocally that halo clustering is a function of halo concentration and show that the dependence of halo bias on concentration, mass, and redshift can be accurately parameterized in a simple way: $b(M,c|z) = b(M|z)b_{c_{\rm vir}}(c|M/M_*)$. Interestingly, the scaling between bias and concentration changes sign with the value of M/M_*: high concentration (early forming) objects cluster more strongly for $M \lesssim M_*$, while low concentration (late forming) objects cluster more strongly for rare high-mass halos, $M \gtrsim M_*$. We show the first explicit demonstration that host dark halo clustering depends on the halo occupation number (of dark matter subhalos) at fixed mass, and discuss implications for halo model calculations of dark matter power spectra and galaxy clustering statistics. The effect of these halo properties on clustering is strongest for early-forming dwarf-mass halos, which are significantly more clustered than typical halos of their mass. Our results suggest that isolated low-mass galaxies (e.g. low surface-brightness dwarfs) should have more slowly-rising rotation curves than their more clustered counterparts, and may have consequences for the dearth of dwarf galaxies in voids. They also imply that self calibrating richness-selected cluster samples with their clustering properties might overestimate cluster masses and bias cosmological parameter estimation.

Subject headings: cosmology: theory — dark matter — galaxies: halos — galaxies: formation — large-scale structure of universe — methods: numerical

1. INTRODUCTION

The spatial distribution of galaxies is now well established to be dependent on several of their internal properties: stellar mass, luminosity, color, star formation rate, Hubble type, and several others (e.g., Hubble 1936; Dressler 1980; Norberg et al. 2001; Zehavi et al. 2002). In the current paradigm for galaxy formation, this can be understood as a combination of the fact that dark matter halos with different masses and formation histories cluster differently and host different galaxy populations. A full understanding of these trends is one of the primary goals of modern cosmology, as it is likely to provide insight into the physical process that govern galaxy formation and aid in the use of observed galaxy clustering as a probe of fundamental cosmological parameters.

Many models of galaxy formation and methods of calculating galaxy clustering statistics make two related simplifying assumptions. The first is that the number and properties of galaxies within a host dark matter halo depend solely on the mass of the host, independent of halo environment or other properties of the dark halo. The second is that the clustering properties of dark matter host halos are a function only of their masses and that dark matter halos are otherwise ignorant of their larger environments. The latter assumption forms part of the basis of the excursion-set formalism for galaxy clustering (Bond et al. 1991), at least in its simplest and most common implementation (Lacey & Cole 1993; Somerville & Kolatt 1999). This implementation assumes that halo formation is a Markov process with no correlations between different spatial scales, which then implies that future halo accretion is independent from past history, and that halo histories are independent of environment (see also discussion in White 1996 and Sheth & Tormen 2004).

The first place that these assumptions are made is in semi-analytic models of galaxy formation. A basic assumption of the technique is that the properties of galaxies depend only on the mass and formation time of the host halo. In many implementations, galaxy clustering is calculated by filling simulated halos at a fixed redshift with halo formation histories that are calculated analytically using the excursion-set formalism (e.g. Kauffmann et al. 1997; Benson et al. 2000; Wechsler et al. 2001; Zentner et al. 2005). By construction, any dependence of galaxy properties on environment in these models must come only from the extent to which they populate halos of different masses. Note that this assumption is avoided to some extent in many modern implementations which use halo merging histories extracted directly from N-body simulations (e.g. Springel et al. 2001; Helly et al. 2003; Hatton et al. 2003; Springel et al. 2005; Kang et al. 2005; Croton et al. 2005; De Lucia et al. 2006;

[1] Kavli Institute for Cosmological Physics, Department of Astronomy & Astrophysics, and Enrico Fermi Institute, The University of Chicago, Chicago, IL 60637 USA
[2] Hubble Fellow
[3] Center for Cosmology, Department of Physics & Astronomy, University of California, Irvine, CA 92697 USA
[4] Physics Department, University of California, Santa Cruz, CA 96050 USA; present address: Pharmix Corp., 2000 Sierra Point Pkwy, Suite 500, Brisbane, CA 94005

Bower et al. 2005). In these implementations semi-analytic recipes depend on mass and formation history as before, but correlations between the galaxy properties and environment may now come from the extent to which they populate halos of different masses *and formation histories*. If there are physical effects that otherwise depend on the larger scale environment these would not be included.

The second place these assumptions are commonly made is in the standard halo model of galaxy clustering (e.g., Seljak 2000; Peacock & Smith 2000; Scoccimarro et al. 2001; Berlind & Weinberg 2001; Bullock, Wechsler, & Somerville 2002; Cooray & Sheth 2002). The halo model is a framework for calculating clustering statistics of objects by associating them with dark matter halos, which have well-studied abundances and clustering properties. The clustering statistics of a galaxy population can be computed after specifying the clustering properties of the dark matter halos, the probability distribution for the number of galaxies in a host halo as a function of halo mass, and the distribution of these galaxies within their host halos. In principle, the halo bias, the probability distribution for the number of galaxies in a halo of fixed mass, and the spatial distribution of galaxies within their host halos can depend on other properties of the halo, but the standard assumption is that they depend only on halo mass. Abbas & Sheth (2005) have recently described a modification of the halo model that incorporates dependencies on local densities.

Lemson & Kauffmann (1999) tested the assumption that halo properties are independent of environment using numerical simulations of cosmological structure formation and found no dependence of halo clustering on formation time or on several other properties of the dark halos. More recent theoretical studies have also indicated that any trends of halo occupation on environment have only a relatively small net effect on large-scale clustering statistics, at least at the level that can be measured in relatively small computational volumes (Berlind et al. 2003; Zentner et al. 2005; Yoo et al. 2005). However, a recent study by Avila-Reese et al. (2005) found environmental trends with halo concentration, spin, shape, and internal angular momentum.

An early indication of a relationship between halo formation histories and halo clustering properties was demonstrated by Sheth & Tormen (2004, see also Wechsler 2001). Recently, Gao, Springel, & White (2005) showed convincingly that the clustering of low-mass halos is a strong function of their formation times. In this mass regime, early-forming halos are significantly more clustered than their late-forming counterparts. Harker et al. (2006) provided confirmation of these results using statistics of marked point distributions similar to those that we employ below. The trend they identified is strongest for low-mass halos, which have only recently been well resolved in numerical simulations.

Many properties of dark matter halos correlate well with halo formation time, so it is natural to investigate whether these trends with formation time extend to other halo properties. In particular, it is natural to expect trends with the concentrations of dark halo density profiles, which Navarro, Frenk, & White (1997), Wechsler et al. (2002), and Zhao et al. (2003) have shown to correlate well with halo formation time. In addition, several studies have convincingly demonstrated that the number of satellite halos within a host halo of fixed mass is a function of halo formation time (Gao et al. 2004; Zentner et al. 2005; van den Bosch et al. 2005; Taylor & Babul 2005) and halo concentration (Zentner et al. 2005). If satellite halos are to be associated with satellite galaxies in groups and clusters, this indicates that the probability distribution for the number of galaxies per halo, known as the halo occupation distribution (HOD), is also a function of these variables and, by extension, may likely be a function of halo environment.

Given the correlations between these halo properties and formation time, it is interesting to determine whether or not they relate to environment in a similar way. This does not have to be the case; if the relations between these halo properties and formation time are themselves a function of environment, their trends with clustering could in principle be quite different. Moreover, as we discuss below, concentration and halo occupation have much more direct consequences for the halo model and its application to constraints on cosmological parameters, and may have a more direct impact on tests of galaxy clustering. We focus our study on these halo properties.

In the present study we use two large, high-resolution dissipationless cosmological simulations to study the dependence of the clustering of dark matter halos on halo properties other than mass. We show that halo clustering depends on halo formation time, and present a clear demonstration that halo clustering is a function of both halo concentration and halo occupation number. We show how these properties change with halo mass, and present the first investigation into how they change with redshift. We present a simple fitting formula for our concentration-dependent clustering results that will enable estimates of the strength of these effects for various applications in the context of the halo model. We discuss several implications of these results for outstanding issues in galaxy formation, including the clustering of dwarf galaxies and the concentrations of low surface-brightness galaxies, and for the estimation of cosmological parameters, including self-calibration of cluster masses.

We begin with a description of our methods in § 2. Specifically, we describe our numerical simulations in § 2.1, and the statistics that we employ in § 2.2. In § 2.3 and § 2.4 we discuss our definitions and measurements of halo concentration and halo formation time respectively. In § 3, we explore the clustering dependence of both halo formation time and halo concentration, and present a model for the relative bias of halos as a function of concentration. In this section we also update previous results on the correlations between both halo formation time and halo concentration and the number of satellite halos contained within a host halo of fixed mass. Following this, we give the first explicit demonstration that host halo clustering is a function of the occupation number of satellite halos. In § 4, we discuss the implications of our results for the halo model. We conclude with a summary of our primary results and a discussion of their implications for galaxy formation models and for cosmological constraints derived from galaxy clustering in § 5.

2. METHODS

2.1. *Numerical Simulations*

We investigate the environmental dependence of halo concentrations and halo occupation using cosmological N-body simulations of structure formation in the concordance, flat ΛCDM cosmology with $\Omega_{\rm M} = 1 - \Omega_\Lambda = 0.3$, $h = 0.7$, and $\sigma_8 = 0.9$. The simulations were performed with the Adaptive Refinement Tree (ART) N-body code (Kravtsov et al. 1997). The two simulations follow the evolution of 512^3 particles in computational boxes of size $120h^{-1}$Mpc and $80h^{-1}$Mpc on a side respectively. We refer to these two simulations as "L120" and "L80." The corresponding particle masses in these simulations are $m_{\rm p} \simeq 1.07 \times 10^9\ h^{-1}{\rm M}_\odot$ in L120 and $m_{\rm p} \simeq 3.16 \times 10^8\ h^{-1}{\rm M}_\odot$ in L80. Both simulations use root computational grids of 512^3 cells and adaptively refine the grids according to the evolving local density fields to a maximum of 8 levels. This results in peak spatial resolutions of $h_{\rm peak} \simeq 1.8h^{-1}$kpc and $h_{\rm peak} \simeq 1.2h^{-1}$kpc in comoving units for L120 and L80 respectively.

We identify halos and subhalos (self-bound halos with centers located within the virial radius of a larger halo) using a variant of the Bound Density Maxima algorithm (BDM, Klypin et al. 1999). Each halo is associated with a density peak, identified using the density field smoothed with a 24-particle SPH kernel. All particles within a search radius of $r_{\rm f} = 25h^{-1}$kpc, set to match the size of the smallest objects we aim to identify, are removed from further consideration as potential halo centers. The BDM algorithm iteratively removes unbound particles from each halo and uses the remaining bound particles to calculate halo properties such as the virial mass $M_{\rm vir}$, circular velocity profile $V_{\rm c}(r) = \sqrt{GM(<r)/r}$, maximum circular velocity $V_{\rm max}$, and the mass within a tidal truncation radius. A more detailed description of the algorithm and specific parameters used is given in Kravtsov et al. (2004a).

We define a virial radius $R_{\rm vir}$, as the radius of the sphere, centered on the density peak, within which the mean density is $\Delta_{\rm vir}(z)$ times the mean density of the universe, $\rho_{\rm M}$. The virial overdensity $\Delta_{\rm vir}(z)$, is given by the spherical top-hat collapse approximation and we compute it using the fitting function of Bryan & Norman (1998). In the ΛCDM cosmology that we adopt for our simulations, $\Delta_{\rm vir}(z = 0) \simeq 337$ and $\Delta_{\rm vir}(z) \to 178$ at $z \gtrsim 1$. In what follows, we use virial mass to characterize the masses of distinct host halos (i.e., halos whose centers do not lie within the virial radius of a larger system). We quantify the sizes of subhalos using their maximum circular velocities, $V_{\rm max}$, because $V_{\rm max}$ is measured more robustly in dense environments and, unlike mass, is not subject to the ambiguity of a particular definition.

2.2. *Correlation Statistics*

We quantify the dependence of clustering on halo properties using the statistics of marked point distributions. For each halo property, or *mark*, quantified by some value m, the distribution of m over all halos may be characterized by the standard one-point statistics, the mean $\langle m\rangle$, the variance $\mathcal{V}(m)$, and higher order moments. To quantify the dependence of clustering upon m one can construct the *mark-correlation function (MCF)* $k_{\rm mm}(r) \equiv \langle m_1 m_2\rangle_{\rm p}(r)/\langle m\rangle^2$, with mark m (Beisbart & Kerscher 2000; see Gottlöber et al. 2002 for another definition; Sheth 2005). The notation indicates that $\langle m_1 m_2\rangle_{\rm p}(r)$ is the average of the product of m_1 and m_2 for halos at points $\vec{x}_1$ and $\vec{x}_2 = \vec{x}_1 + \vec{r}$ in pairs separated by distance $r = |\vec{r}|$. Similarly, one can compute the average value of m, $\langle m\rangle_{\rm p}(r)$, on the condition that a halo is part of a pair at separation r. This formalism can easily be extended to discrete marks, like Hubble type or number of satellite halos (e.g., Beisbart & Kerscher 2000).

Values of $k_{\rm mm}(r) > 1$ indicate preferred clustering of halos with m higher than average. The magnitude of deviations of $k_{\rm mm}(r)$ from 1 is set by the size of fluctuations of m. In what follows, we employ a modified MCF that is normalized to the intrinsic one-point fluctuations in m, namely

$$\mathcal{M}_m(r) \equiv (\langle m_1 m_2\rangle_{\rm p}(r) - \langle m\rangle^2)/\mathcal{V}(m). \qquad (1)$$

In the absence of spatial segregation on m, $\langle m_1 m_2\rangle_{\rm p}(r) = \langle m\rangle^2$ and $\mathcal{M}_m(r) = 0$. The deviations from the case of no segregation are expressed in units of $\mathcal{V}(m)$. Roughly speaking, a value of $\mathcal{M}_m(r) = 0.25$ is indicative that halos in a pair separated by distance r have values of m that are 0.5σ $(= \sqrt{0.25}\sigma)$ higher than $\langle m\rangle$.

Beisbart & Kerscher (2000), Gottlöber et al. (2002), and more recently, Sheth, Connolly, & Skibba (2005), have used MCFs to study luminosity- and morphology-dependent clustering in observational, simulated, and semi-analytic samples, respectively. Sheth & Tormen (2004) and Harker et al. (2006) used the statistics of marked point distributions to study the dependence of halo clustering on mass assembly history in cosmological simulations. In this paper, we apply mark-correlation functions to study the environmental dependence of halo concentrations and subhalo abundance.

2.3. *Halo Concentrations*

The spherically-averaged density profiles of cosmological halos can be described by the profile of Navarro, Frenk, & White (1997, hereafter NFW)

$$\rho(r) = \rho_0 (r/r_{\rm s})^{-1}(1 + r/r_{\rm s})^{-2}. \qquad (2)$$

The transition radius between the inner and outer power laws $r_{\rm s}$, is often quantified by the concentration parameter $c_{\rm vir} \equiv R_{\rm vir}/r_{\rm s}$. In the next section, we quantify the dependence of halo clustering on $c_{\rm vir}$. We assign each halo a best-fit concentration by fitting halo density profiles in logarithmically-spaced radial bins following Bullock et al. (2001, B01 hereafter). We consider only halos with more than 250 particles within their virial radii, resulting in lower mass limits of $M_{\rm min} \simeq 2.7\times10^{11}\ h^{-1}{\rm M}_\odot$ for L120 and $M_{\rm min} \simeq 7.9\times10^{10}\ h^{-1}{\rm M}_\odot$ for L80. In addition to the L120 and L80 simulations, we re-analyzed the simulation of B01 for the mean $c_{\rm vir}$-$M_{\rm vir}$ relation. The B01 simulation had the same formal spatial and mass resolution as L120 in a computational box of $60h^{-1}$Mpc on a side. The cosmology adopted in B01 was a ΛCDM cosmology with a power spectrum normalization of $\sigma_8 = 1.0$. Hereafter, we refer to this simulation as "L60." We have reanalyzed the L60 simulation and reproduced the $c_{\rm vir}(M_{\rm vir})$ results of B01 using our techniques.

The mean relation for $c_{\rm vir}(M_{\rm vir})$ in the L80 and L120 simulations is well described by the model of B01 *with modified model parameters.* The scatter in $c_{\rm vir}$ at fixed

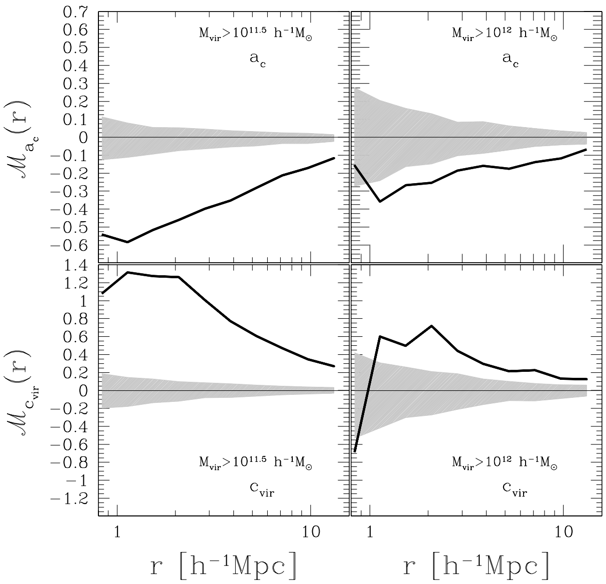

FIG. 1.— Clustering and halo properties at $z = 0$. The top panels show MCFs $\mathcal{M}_{a_c}(r)$ (*solid lines*), with normalized formation time $\widetilde{a}_c$, as mark for two different mass cuts, $M_{\rm vir} \geq 10^{11.5}\ h^{-1}{\rm M}_\odot$ ($\geq$ 300 particles) and $M_{\rm vir} \geq 10^{12}\ h^{-1}{\rm M}_\odot$ ($\geq$ 940 particles). The *shaded bands* represent the 95$^{\rm th}$ percentile of $\mathcal{M}_{a_c}(r)$ formed from 200 random reassignments of the marks to halos in the sample. The bottom panels show MCFs $\mathcal{M}_{c_{\rm vir}}(r)$ with normalized concentration $\widetilde{c}_{\rm vir}$ as the mark at the same host halo mass thresholds. The lines and the shaded band have the same significance as the top panels.

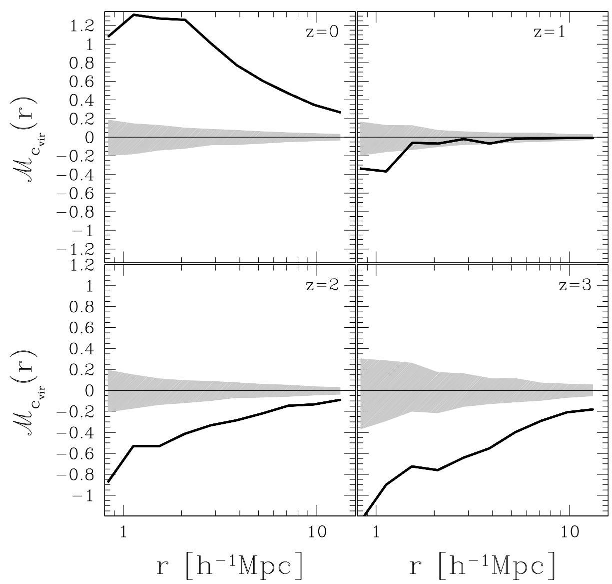

FIG. 2.— The dependence of clustering on $c_{\rm vir}$ as a function of redshift. The panels show MCFs $\mathcal{M}_{c_{\rm vir}}(r)$ (*solid lines*) with normalized halo concentration $\widetilde{c}_{\rm vir}$, as mark for halos with $M_{\rm vir} \geq 10^{11.50}\ h^{-1}{\rm M}_\odot$ at four different redshifts. *Shaded bands* represent the 95$^{\rm th}$ percentile of $\mathcal{M}_{c_{\rm vir}}(r)$ from 200 random reassignments of the marks.

halo mass is well described for both simulations by a log-normal distribution with a standard deviation of $\sigma(\log c_{\rm vir}) \simeq 0.14$ at all probed masses in accordance with B01 and Wechsler et al. (2002). In the notation of B01, we find that our $c_{\rm vir}(M_{\rm vir})$ relation is well described by the B01 model with $F = 10^{-3}$ and $K = 2.9$. The revised parameters result in a shallower scaling of concentration with mass, and slightly lower values of the concentration around M_*. It also predicts concentration values that are about 20% lower at $10^{11}\ h^{-1}{\rm M}_\odot$, but note that this is still an extrapolation of the model to lower mass scales than it has been measured robustly by our simulations. A similar revision of the B01 parameters was previously proposed by Dolag et al. (2004, see also Kuhlen et al. 2005 who advocated lower K) to accommodate their simulations of cluster-sized halos and is in broad agreement with the profiles of observed clusters (Vikhlinin et al. 2006). The differences in the $c_{\rm vir}(M_{\rm vir})$ relation are attributable to a combination of the differences in the initial power spectra of the simulations and cosmic variance due to the finite sizes of the computational volumes.

2.4. *Halo Formation Times*

For each halo in our sample, we have determined a mass accretion history $M_{\rm vir}(a)$, by identifying the most massive progenitor of each halo as a function of time using an algorithm similar to that of Wechsler et al. (2002, more details are given in Allgood 2005). Wechsler et al. (2002) found that the halo mass accretion histories can be characterized by a one-parameter family of trajectories of the form

$$M(a) = M_{\rm o}\exp\left[-2a_{\rm c}\left(\frac{a_{\rm o}}{a} - 1\right)\right], \quad (3)$$

where $a_{\rm o}$ and $M_{\rm o}$ are the scale factor and mass at the time the halo is observed. Equation (3) defines a formation scale factor $a_{\rm c}$. We assign to each halo a value of $a_{\rm c}$ according to the value that best fits its mass accretion history, following Wechsler et al. (2002). In the following section, we address halo clustering as a function of $a_{\rm c}$.

The formation time $a_{\rm c}$ has a number of advantages over other definitions of halo formation times, such as the times when halos first acquire fixed fractions of their final mass. The quantity $a_{\rm c}$ is less sensitive to individual events in the formation of a halo, as it is based on the entire mass accretion history of each halo, rather then a single epoch. As shown by Wechsler et al. (2002), it also has the property that its distribution and average value are only a function of mass, and not redshift.

Wechsler et al. (2002) showed that the formation time $a_{\rm c}$ is tightly correlated with $c_{\rm vir}$: the mean relation given by $c_{\rm vir} = c_1/a_{\rm c}$, where c_1 is the concentration of halos forming today. In the ΛCDM cosmology adopted in our simulation, these correspond to halos with $M \sim 10^{15}\ h^{-1}{\rm M}_\odot$, and $c_1 \sim 4$ (see Wechsler et al. 2002 for details). At fixed redshift, $a_{\rm c}(M_{\rm vir})$ is a weak function of $M_{\rm vir}$ and the distribution of $a_{\rm c}$ at fixed mass can be characterized by a log-normal distribution with $\sigma(\log a_{\rm c}) = \sigma(\log c_{\rm vir}) \simeq 0.14$ (Wechsler et al. 2002).

3. RESULTS

3.1. *Formation Time and Concentration Marks*

Figure 1 shows mark-correlation functions with halo formation time and concentration used as marks. The simple mass dependence of $a_{\rm c}$ and $c_{\rm vir}$, and the simple distributions of these quantities at fixed mass allow us to

scale out the gross mass dependence of these quantities in studying formation time- and concentration-dependent clustering. We accomplish this by assigning each halo a normalized formation time $\widetilde{a}_{\rm c} \equiv a_{\rm c}/\langle a_{\rm c}(M_{\rm vir})\rangle$ and concentration $\widetilde{c}_{\rm vir} \equiv c_{\rm vir}/\langle c_{\rm vir}(M_{\rm vir})\rangle$, where $\langle a_{\rm c}(M_{\rm vir})\rangle$ and $\langle c_{\rm vir}(M_{\rm vir})\rangle$ are the averages of formation time and concentration as a function $M_{\rm vir}$ computed in bins of width $\Delta\log(M_{\rm vir}) = 0.20$.

Consider first the top panels of Figure 1, where we show MCFs $\mathcal{M}_{a_{\rm c}}(r)$ with mark $\widetilde{a}_{\rm c}$, for distinct halos above two mass thresholds, $M_{\rm vir} \geq 10^{11.5}\ h^{-1}{\rm M}_\odot$ and $M_{\rm vir} \geq 10^{12}\ h^{-1}{\rm M}_\odot$. We represent the statistical significance of deviations of the MCF from the null hypotheses of the absence of spatial segregation on formation time or concentration by randomly reassigning the marks among the halos in the sample 200 times and recomputing the MCFs on these random samples. The shaded regions in Figure 1 and all MCF figures that follow show the envelope formed by 95% of these randomized MCFs.

Figure 1 shows a statistically-significant tendency for early-forming (low $a_{\rm c}$) halos to be more strongly clustered in both mass bins. The strength of the trend diminishes with increasing mass. These results are in qualitative agreement with Gao et al. (2005) and Harker et al. (2006), who define formation time as the time the halo first acquired half of its final mass. We find a similar signal for this definition of formation time as well. This figure also clearly shows a dependence with scale, with stronger trends at a few h^{-1}Mpc than on larger scales (note that the spread in the range for randomized samples indicates larger errors at small radii due to fewer pairs of halos in these bins). In what follows, we explore the redshift dependence of clustering as a function of concentration $c_{\rm vir}$, because robust determinations of $a_{\rm c}$ become increasingly difficult at high redshift as smaller portions of the halo mass accretion histories are sampled.

The known relationship between formation time and $c_{\rm vir}$ suggests that halo clustering should be a strong function of $c_{\rm vir}$ unless the relationship between $c_{\rm vir}$ and $a_{\rm c}$ is itself a strong function of environment. Neither Lemson & Kauffmann (1999), looking for trends with density, nor Sheth & Tormen (2004), using mark-correlation statistics, were able to detect any significant clustering segregation with concentration. However, the simulations employed in these studies were not particularly well suited to resolve the detailed density structures of halos, especially at low mass where the trends are strongest. Our simulations cover a similar computational volume, but they have substantially higher mass and force resolutions compared with these earlier studies.

Figure 1 clearly shows a tendency for preferential clustering of halos selected by their concentrations. In the bottom panels, we show $\mathcal{M}_{c_{\rm vir}}(r)$ at $z = 0$ for host halos above two mass thresholds. As might have been expected based on the $a_{\rm c}$-dependent clustering, halos with high concentrations are more strongly clustered than average. As was the case for formation times, the strength of the $c_{\rm vir}$-dependent clustering is striking at the lowest masses. Halos with $M_{\rm vir} \geq 10^{11.5}\ h^{-1}{\rm M}_\odot$ in pairs separated by $\lesssim 3\ h^{-1}$Mpc tend to have values of $c_{\rm vir}$ more than $\sim 1\sigma$ above the mean relation. The statistical significance of this preferential clustering persists to separations $r > 10\ h^{-1}$Mpc, where halos in pairs have $c_{\rm vir}$ values $\gtrsim 0.5\sigma$ above the mean. Just as with $a_{\rm c}$, the $c_{\rm vir}$-dependent clustering is a decreasing function of halo mass over this range. The figure indicates that in our simulation, the dependence of clustering on concentration is even stronger than the dependence of clustering on halo formation time. We have verified this by remaking the bottom half of Figure 1 with the variable $c_{ac} \equiv c_1/a_c$ as the mark. This plot looks quite similar to the inverse of the formation time mark, and does not show as strong of a signal as the measured concentration. It seems likely that this discrepancy is just due to larger measurement errors in formation time, but a more detailed analysis with a larger simulation will be necessary to determine this.

We explore $c_{\rm vir}$-dependent clustering at fixed virial mass ($M_{\rm vir} > 10^{11.5} M_\odot$) as a function of redshift in Figure 2. Interestingly, this effect is a strong function of redshift. Indeed the sense of the clustering trend reverses over the redshift range shown here. Above this fixed absolute mass threshold, high-concentration halos are more strongly clustered at $z = 0$, while at $z \sim 1$ there is, at most, a weak trend and at $z \gtrsim 2$ *low-concentration* halos tend to be clustered more strongly. This tendency for halos with low $c_{\rm vir}$ values to be more weakly clustered at high redshift increases steadily with redshift thereafter. This trend suggests that late-forming (high-$a_{\rm c}$) halos are actually more strongly clustered at high redshift though we show no direct statistically-significant evidence of this, largely due to the difficulty in making robust determinations of $a_{\rm c}$ at high redshift.

3.2. *Concentration-Dependent Halo Bias*

Due to resolution requirements for measuring $c_{\rm vir}$, we sample halos above a fixed mass at each redshift in order to compute the MCFs in Figure 2, but the typical collapsing mass $M_\star$, is a declining function of z. At $z = 0$, $M_\star \simeq 8.4 \times 10^{12}\ h^{-1}{\rm M}_\odot$, while by $z = 2$, $M_\star \simeq 1.9 \times 10^{10}\ h^{-1}{\rm M}_\odot$, so objects at fixed mass become increasingly rare with increasing z. It is natural to suspect that the mass and redshift dependence found in Figures 1 and 2 may have a common origin, due to $c_{\rm vir}$- or $a_{\rm c}$-dependent clustering that is a function of the relative rarity of the peaks from which these halos form in the primordial density field (e.g., Mo & White 1996). To test this hypothesis, we explore the relative clustering of halos as a function of concentration and scaled mass $M_{\rm vir}/M_\star$.

We are unable to test this mass scaling over a large dynamic range at a single redshift due to the limited dynamic range of our simulations. In order to explore this $M_{\rm vir}/M_\star$ scaling we use the L80 simulation at $z = 0$ to explore the low-$M_{\rm vir}/M_\star$ regime and we use several timesteps from the L120 simulation to probe higher values of $M_{\rm vir}/M_\star$. To quantify $c_{\rm vir}$-dependent clustering, we define a *relative* bias for a subsample of halos compared to all halos in the same mass range,

$$b^2_{c_{\rm vir}}(r|\widetilde{m}) = \xi_{\rm subsample}(r|\widetilde{m})/\xi_{\rm all}(r|\widetilde{m}), \qquad (4)$$

where we have defined a scaled mass variable $\widetilde{m} \equiv M_{\rm vir}/M_\star$. For each subsample, $b_{c_{\rm vir}}(r)$ is consistent with being constant over $5 \leq r/\ h^{-1}{\rm Mpc} \leq 10$, so we reduce this function to one number: $b^2_{c_{\rm vir}}$ taken over the range of halo separations $r = 5{-}10\ h^{-1}$Mpc. In Figure 3, we com-

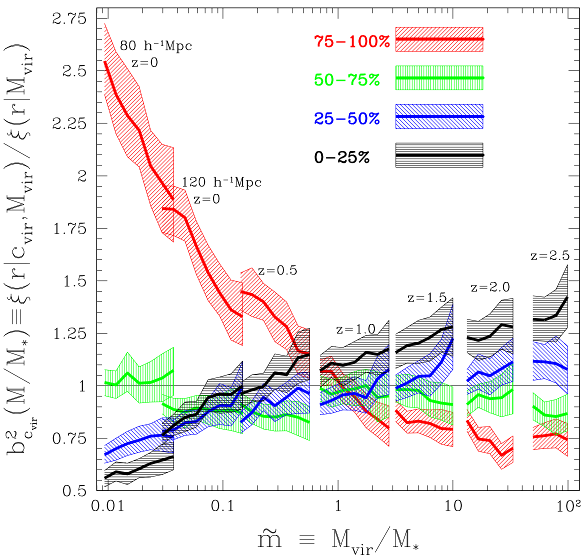

FIG. 3.— Relative bias squared for halo samples selected by quartiles in $\tilde{c}_{\rm vir}$ and thresholds in the mass variable $\tilde{m}$, compared to the bias of all halos above the same mass threshold. Each set of curves shows the mean bias for the indicated $\tilde{c}_{\rm vir}$ quartile. The shaded bands represent the 68% region constructed from 200 random subsamples of the unbiased population with the same size as the biased subsample. The leftmost segments are taken from the $z = 0$ output of the L80 simulation and are labeled by "80 h^{-1}Mpc". The remaining segments are taken from different redshift outputs of the L120 simulation (labeled "120 h^{-1}Mpc") as indicated in order to fill in the entire range of $M_{\rm vir}/M_\star$. The left edge of each segment is determined by a minimum of 250 particles in a halo, while the right edge is limited by requiring that there be more than 1500 halos in each subsample.

pare this relative bias as a function of scaled halo mass $b^2_{c_{\rm vir}}(\tilde{m})$ for several subsamples selected on percentiles of $\tilde{c}_{\rm vir}$. The shaded bands account for the measurement error in $b^2_{c_{\rm vir}}$ due to sub-sampling the full halo distribution by recomputing $\xi_{\rm all}(r)$ from 200 random subsamples of the total halo population with the same number of objects as contained in each subsample. The bands represent the contours containing 68% of the $b^2_{c_{\rm vir}}$ values computed in this manner. Different ranges in $M_{\rm vir}/M_\star$ are covered by simulation outputs at different redshifts as labeled in the figure. Scaling the results by $M_\star$ delineates a well-defined trend in this concentration bias as a function of $\tilde{m}$. In each redshift range, we are limited at the low-mass end by resolution; we require that a halo have at least 250 particles within its virial radius in order to be considered. At the high-mass end, the bands are limited by the requirement that there be at least 1500 halos in each subsample. These requirements give rise to the finite length of each segment in Figure 3. Results from the L80 simulation at $z \sim 1$ are in good agreement with the L120 simulation at $\tilde{m} \sim 0.1$; however, we do not plot these in the interest of clarity.

Figure 3 clearly demonstrates the trend already indicated by the mark-correlation functions for the highest-$c_{\rm vir}$ halos to be much more strongly clustered than average for $M_{\rm vir} \lesssim M_\star$ and less strongly clustered than the overall halo population for $M_{\rm vir} \gtrsim M_\star$. It is worth noting at this point that above $M_\star$, where the scaling of bias with mass is very strong, mass is still the dominant variable in determining bias. However, well below $M_\star$, the scaling of bias with mass flattens, and formation time appears to be the dominant variable determining bias.

Below, we provide a fitting function for the bias as a function of both concentration and mass. Our simulation data are not sufficient to determine this function with high accuracy, but we present this function to give a convenient way to estimate the magnitude of the effects of these trends in particular applications such as the clustering of specific galaxy populations.

Let $c' \equiv \ln(\tilde{c}_{\rm vir})/\sigma(\ln c_{\rm vir}) = \log(\tilde{c}_{\rm vir})/\sigma(\log c_{\rm vir})$, such that the probability distribution of c' at fixed halo mass $P(c'){\rm d}c'$ is Gaussian with unit variance. We define the relative bias of halos as a function of c', as the ratio of the clustering amplitude of halos of fixed $\tilde{m}$ and fixed c' relative to the clustering amplitude of all halos of fixed $\tilde{m}$,

$$b^2_{c_{\rm vir}}(c'|\tilde{m}) \equiv \frac{\xi(r, c'|\tilde{m})}{\xi(r|\tilde{m})}, \tag{5}$$

where here we have again taken the average of the halo bias over separations from $5 \le r/\ h^{-1}{\rm Mpc} \le 10$. We find that a good fit to the simulation data is given by

$$b_{c_{\rm vir}}(c'|\tilde{m}) = p(\tilde{m}) + q(\tilde{m})c' + 1.61[1 - p(\tilde{m})]c'^2, \tag{6}$$

where

$$p(\tilde{m}) = 0.95 + 0.042\ln(\tilde{m}^{0.33})$$
$$q(\tilde{m}) = 0.1 - \frac{0.22[\tilde{m}^{0.33} + \ln(\tilde{m}^{0.33})]}{[1 + \tilde{m}^{0.33}]}.$$

The best fitting parameters for this relation satisfy the normalization condition

$$\int b_{c_{\rm vir}}(c'|\tilde{m})P(c'){\rm d}c' = 1.0 \tag{7}$$

to within a few percent. The simulation results are consistent with this normalization within the sizable errors, and we justify this constraint in more detail in § 4, where we discuss the implications of this relative bias of halos on the halo model. In Figure 4, we show the fit of Eq. (6) compared to the relative bias measured from the simulations as a function of the scaled concentration variable c', for several values of the scaled halo mass $\tilde{m}$.

3.3. *Halo Occupation Mark*

We expect that this clustering effect may extend to other properties of halos and the galaxies they host, especially those which are known to be strongly correlated with formation time and halo structure. Halo angular momentum and halo shape are two such halo properties that are relevant to galaxy formation and known to correlate well with halo formation history (e.g., Vitvitska et al. 2002; Allgood et al. 2006)

The quantity from dissipationless simulations that is most pertinent to models of the statistics of galaxy clustering is $P(N_{\rm sat}|M_{\rm vir})$, the probability distribution of the number of subhalos per host halo at fixed host halo mass. In such simulations this is the best proxy for the number distribution of satellite galaxies per halo (e.g., Kravtsov et al. 2004a). The probability distribution of this number of satellite galaxies per halo as a function of halo mass, $P(N_{\rm sat}|M_{\rm vir})$ is a primary ingredient in halo

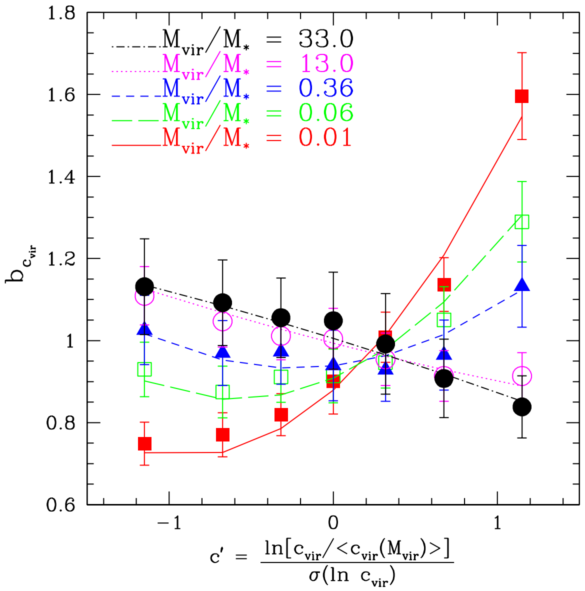

FIG. 4.— Relative bias as a function of normalized concentration, $c' \equiv \log(\widetilde{c}_{\rm vir})/\sigma(\log \widetilde{c}_{\rm vir})$, for various values of halo mass scaled by the typical collapsing mass $\widetilde{m} \equiv M_{\rm vir}/M_{\star}$. The *points* show the values of the relative bias, $b_{c_{\rm vir}}$, measured directly from the host halos in the L120 and L80 simulations and the *lines* show the fit of Eq. (6). We show the relative bias at five different scaled halo masses, $\widetilde{m} = 33$, $\widetilde{m} = 13$, $\widetilde{m} = 0.36$, $\widetilde{m} = 0.06$, and $\widetilde{m} = 0.01$.

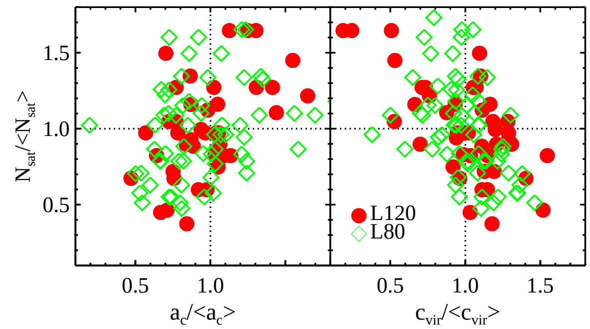

FIG. 5.— Correlation between halo occupation number and formation scale factor (*left panel*) and concentration (*right panel*), counting subhalos 1000 times less massive than their hosts. Host halos more massive than $1 \times 10^{14}\ h^{-1}{\rm M}_{\odot}$ are plotted from the L120 simulation (*red circles*), and host halos more massive than $5 \times 10^{13}\ h^{-1}{\rm M}_{\odot}$ are plotted from the L80 simulation (*green diamonds*). Each of the variables is normalized to the mean of the variable as a function of halo mass.

model calculations of galaxy clustering (see § 4 below). Sheth & Tormen (2004) and Gao et al. (2005) have emphasized that the formation time dependence of clustering breaks a fundamental assumption of the halo model, namely that galaxies populate halos of a given mass in a manner that is statistically independent of halo environment. In fact, this is true only if $P(N_{\rm sat}|M_{\rm vir})$ is a function of halo formation time. Subhalos are natural sites for galaxy formation, so a more direct test is to show that halos cluster differently as a function of $N_{\rm sat}$.

Zentner et al. (2005) showed that both $a_{\rm c}$ and $c_{\rm vir}$ are strongly correlated with $N_{\rm sat}$ in host halos of fixed mass. We update this correlation for the massive halos in the L120 and L80 simulations in Figure 5, where we compare the number of satellites with $M_{\rm host} > 10^3 M_{\rm sub}$ in the massive host halos of the L120 and L80 simulations with the host halo concentrations and formation times. Scaling the satellite number with respect to the host mass normalizes out the gross dependence of satellite number on host halo mass. Moreover, we have normalized both $c_{\rm vir}$ and $a_{\rm c}$ to their average values as a function of halo mass. Figure 5 clearly shows that early-forming, high-concentration halos have fewer satellites. The basic reason is that halos that accrete their subhalos first have more time for those subhalos to be destroyed or to merge with the central object due to dynamical friction (e.g., Kravtsov & Klypin 1999; Taffoni et al. 2003; Zentner & Bullock 2003; Zentner et al. 2005; van den Bosch et al. 2005; Taylor & Babul 2005).

In light of this strong correlation, the clustering dependence of formation time and halo concentration found in the previous section suggests that halo clustering is likely to be a function of $N_{\rm sat}$ as well. Kravtsov et al. (2004a), Tasitsiomi et al. (2004), and Conroy, Wechsler, & Kravtsov (2006) have demonstrated that halos and subhalos selected by their maximum circular velocities provide excellent matches to the observed galaxy-galaxy autocorrelation function, galaxy-mass cross correlation function, as well as to the luminosity-dependence and redshift evolution of clustering, respectively (see also Berrier et al. 2006 for a similar result for close-pair statistics). Following these studies and the arguments in § 2.1 for quantifying subhalo size as a function of maximum circular velocity, we study host halo clustering as a function of the number of satellites above a $V_{\rm max}$ threshold as a quantity that is particularly relevant to galaxy clustering predictions.

In Figure 6 we make a first attempt to quantify the strength of clustering for samples of halos marked by their occupation number. For two halo samples, we have selected subhalos with maximum circular velocities $V_{\rm sat}$, above a fixed fraction of the maximum circular velocities of their hosts $V_{\rm host}$. Taking this ratio of circular velocities scales out the dependence of $N_{\rm sat}$ on host halo size. Figure 6 shows a relatively small but statistically-significant tendency for halos in pairs separated by $\sim 5-10\ h^{-1}$Mpc to have above-average numbers of satellites. This is the most direct demonstration yet that the halo occupation by galaxies is a function of environment.

Note that in Figure 6, we are forced to study only large host halos in order to guarantee that their subhalos are well resolved. As such, we probe a different range of host halo masses than shown in Figure 2. The sense of the trend is what we would expect at this mass range, which is slightly bigger than $M_{\star}$, from the correlation between a_c and $N_{\rm sat}$. Low-concentration, late-forming halos in this mass range are more clustered than average, and it is these halos that are expected to have more satellites. Based on the previous results for $a_{\rm c}$- and $c_{\rm vir}$-dependent clustering, if one were able to measure the clustering of low-mass halos with several satellites, this trend may reverse.

4. PROPERTY-DEPENDENT HALO CLUSTERING AND THE HALO MODEL

The results of the previous two sections have potentially important implications for the halo model of clus-

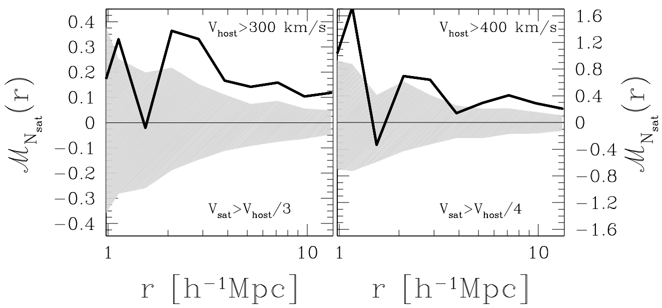

FIG. 6.— The dependence of clustering on satellite number. The panels show MCFs $\mathcal{M}_{N_{\rm sat}}(r)$ (*solid lines*), with the number of dark matter subhalos $N_{\rm sat}$, as mark. The two panels correspond to different samples. In the *left panel*, results are shown for all host halos with maximum circular velocities $V_{\rm host} \geq 300\ {\rm km\,s^{-1}}$ and satellites with $V_{\rm sat} \geq V_{\rm host}/3$. In the *right panel*, we show a sample with $V_{\rm host} \geq 400\ {\rm km\,s^{-1}}$ and $V_{\rm sat} \geq V_{\rm host}/4$. In each panel, the *shaded bands* have the same meaning as in Figure 2.

tering. The basic idea behind the halo model framework has a long history, initially in analytic models that described galaxy clustering as a superposition of randomly-distributed clusters with specified profiles and a range of cluster masses (Neyman & Scott 1952; McClelland & Silk 1977; Peebles 1974). The explosion of recent activity in this field has been partly fueled by the recognition that a combination of this approach with recently developed tools for predicting the spatial clustering of dark matter halos (e.g., Mo & White 1996; Sheth & Tormen 1999; Sheth et al. 2001b; Seljak & Warren 2004; Tinker et al. 2005) provides a powerful formalism for analytic calculations of dark matter clustering, which can be extended naturally to biased galaxy populations.

In modern implementations of the halo model (e.g., Scherrer & Bertschinger 1991; Seljak 2000; Ma & Fry 2000; Peacock & Smith 2000; Scoccimarro et al. 2001) two-point galaxy clustering is calculated by specifying the clustering of dark matter, the non-linear clustering of dark matter halos, the first two moments of the HOD, and the spatial distribution of galaxies within their host halos. The standard implementation also assumes that *halo clustering is independent of all halo properties aside from halo mass.* In particular, in calculations of either galaxy or dark matter correlation functions, it is assumed that the HOD and halo concentrations depend only on halo mass and that there is no spatial segregation of halos based on their occupation numbers, concentrations, or any other properties that may be relevant to the properties of the galaxies that the halos host. We refer to a halo model based on this set of assumptions as the "strong" halo model. As we have shown, these assumptions are not generically valid: the two-point clustering of dark matter halos depends on halo concentration, halo formation time, and halo occupation. Below, we review several salient aspects of the halo model and explore the implications of relaxing these assumptions about the lack of environmental dependence of halo properties. As an illustrative example that is closely tied to the results of the previous section, we focus most of our attention on relaxing the assumption that clustering is independent of the halo profile concentration. It is worth keeping in mind that there are several different implementations of various aspects of the halo model: choices must be made about how to model the translinear regime for galaxies (especially the treatment of halo exclusion and scale-dependent bias, see e.g. Tinker et al. 2006, Appendix B), what analytic models to use for the mass function and bias, and how to model the halo occupation (Zheng 2004; Conroy et al. 2006, e.g.) and the profiles of galaxies in halos.

4.1. *The Dark Matter Correlation Function in the Standard Halo Model*

The simplest application of the halo model is to calculate the dark matter two-point correlation function. The halo model breaks the computation into a "one-halo" term receiving contributions from the mass density in individual halos on small scales and a "two-halo" term with contributions from mass in distinct pairs of halos on large scales. In the standard halo model, we can write the large-scale halo correlation function of dark matter as (Scherrer & Bertschinger 1991):

$$\xi_{\rm dm}(r) = \xi^{\rm 1h}(r) + \xi^{\rm 2h}(r) + 1. \tag{8}$$

The one-halo term is

$$\xi^{\rm 1h}(r) = \frac{1}{\rho_{\rm M}^2}\int {\rm d}m\, m^2 \frac{{\rm d}n(m)}{{\rm d}m}\int {\rm d}^3x\, \lambda_m(\vec{x})\lambda_m(\vec{x}+\vec{r}), \tag{9}$$

with ${\rm d}n(m)/{\rm d}m$ the mass function of halos and $\lambda_m(\vec{x})$ the density distribution within a halo of mass m normalized so that the integral of the profile over the volume of the halo is unity. The two-halo term is

$$\begin{aligned}\xi^{\rm 2h}(r) =& \frac{1}{\rho_{\rm M}^2}\int {\rm d}m_1 \int {\rm d}m_2\, m_1 \frac{{\rm d}n(m_1)}{{\rm d}m_1}\, m_2 \frac{{\rm d}n(m_2)}{{\rm d}m_2}\\ &\times \int {\rm d}^3x \int {\rm d}^3y\ \lambda_{m_1}(\vec{x})\lambda_{m_2}(\vec{y})\\ &\times\ \xi_{\rm hh}(\vec{x}-\vec{y}+\vec{r}|m_1,m_2),\end{aligned} \tag{10}$$

where $\xi_{\rm hh}(\vec{x}|m_1,m_2)$ is the cross-correlation function of halos of mass m_1 and m_2 and $r \equiv |\vec{r}|$.

In the limit of separations much larger than the sizes of the largest halos, the correlation function is determined by the two-halo term alone. On such large scales, the correlation functions vary little over the length scales of halos so that $\xi_{\rm hh}(\vec{x}-\vec{y}+\vec{r}|m_1,m_2) \simeq \xi_{\rm hh}(\vec{r}|m_1,m_2)$, which allows the last two integrals in Eq. (10) to be replaced by $\xi_{\rm hh}(\vec{r}|m_1,m_2)$. Relating the halo correlation functions to the dark matter correlation function through the standard assumption $\xi_{\rm hh}(r|m_1,m_2) \simeq b_{\rm h}(m_1)b_{\rm h}(m_2)\xi_{\rm dm}(r)$ and requiring $\xi^{\rm 2h}(r) = \xi_{\rm dm}(r)$ on large scales forces the halo bias $b_{\rm h}(m)$ to obey the constraint

$$\int {\rm d}m\, \frac{{\rm d}n(m)}{{\rm d}m}\left(\frac{m}{\rho_{\rm M}}\right) b_{\rm h}(m) = 1. \tag{11}$$

This is the well-known normalization rule for the mass-dependent halo bias.

4.2. *The Galaxy Correlation Function in the Halo Model*

This model can be used to compute the statistics of any population for which all members reside in dark matter

halos. The most popular application is to compute the correlation statistics of galaxies. The equations of § 4.1 can be adapted to this application simply by making the following substitutions. First, take $\rho_{\rm M}^2 \to \overline{n}_{\rm g}/2$, where $\overline{n}_{\rm g}$ is the mean number density of galaxies. Take $m \to \langle N_{\rm gal}\rangle_m$ so that the mean number of galaxies per halo of mass m is counted rather than the mass per halo. Take $m^2 \to \langle N_{\rm gal}(N_{\rm gal}-1)\rangle_m/2$ so that pairs of galaxies within halos of mass m are counted. Finally, take $\lambda_m(\vec{x}) \to \lambda_m^g(\vec{x})$ to represent the the mean distribution of galaxies within host halos of mass m rather than the distribution of mass within halos.

Following the logic of the previous section, we can compute the large-scale clustering of galaxies from the two-halo term alone. This leads to the well-known and useful relation for the large-scale bias of a galaxy population given the first moment of its HOD, $\langle N_{\rm gal}\rangle_m$,

$$b_{\rm gal} \simeq \frac{1}{\overline{n}_{\rm g}} \int {\rm d}m\, \frac{{\rm d}n(m)}{{\rm d}m} \langle N_{\rm gal}\rangle_m b_{\rm h}(m)\,. \tag{12}$$

As one might expect, the large-scale bias of galaxies is given by a simple, weighted average of the bias of the halos in which they reside.

4.3. *The Dark Matter Correlation Function with $c_{\rm vir}$-Dependent Halo Clustering*

Now consider recasting the halo model allowing halo clustering to be a function of both halo mass m, and the additional property of concentration c. If we define the probability of a value of c at fixed mass as $P(c|m)$ then the number of halos of mass m with concentration c is ${\rm d}n(m,c)/{\rm d}m{\rm d}c = P(c|m){\rm d}n(m)/{\rm d}m$. This additional property complicates the halo model and requires an extra integral over the distribution of halo concentrations. Specifically, the one- and two-halo terms become

$$\begin{aligned}\xi^{\rm 1h}(r) = &\frac{1}{\rho_{\rm M}^2} \int {\rm d}m \int {\rm d}c\, m^2 P(c|m) \frac{{\rm d}n(m)}{{\rm d}m} \\ &\times \int {\rm d}^3x \lambda_m(\vec{x}|c)\lambda_m(\vec{x}+\vec{r}|c)\end{aligned} \tag{13}$$

and

$$\begin{aligned}\xi^{\rm 2h}(r) = &\frac{1}{\rho_{\rm M}^2} \int {\rm d}m_1 \int {\rm d}c_1 \int {\rm d}m_2 \int {\rm d}c_2 \\ &\times m_1 \frac{{\rm d}n(m_1)}{{\rm d}m_1} P(c_1|m_1)\; m_2 \frac{{\rm d}n(m_2)}{{\rm d}m_2} P(c_2|m_2) \\ &\times \int {\rm d}^3x \int {\rm d}^3y \lambda_{m_1}(\vec{x}|c_1)\, \lambda_{m_2}(\vec{y}|c_2) \\ &\times \xi_{\rm hh}(\vec{x}-\vec{y}+\vec{r}|m_1,c_1,m_2,c_2)\,,\end{aligned} \tag{14}$$

where $\lambda_m(\vec{x}|c)$ is the density distribution for a halo of mass m and concentration c, and where the cross correlation of halos of mass m_1 and concentration c_1 with halos of mass m_2 and concentration c_2 is embodied in $\xi_{\rm hh}(\vec{x}|m_1,c_1,m_2,c_2)$.

Two consequences of these relations are evident. The first is simply that the one-halo term at small separations should be a weighted average of profile convolutions with the weighting given by $P(c|m)\,{\rm d}n(m)/{\rm d}m$. This is true independent of concentration-dependent clustering has been overlooked in most modeling efforts (although, see e.g. Sheth et al. 2001a). The effect of adding scatter is only at the few percent level, but this will be important for future precision measurements of dark energy using lensing as the statistical uncertainty of the experiments approaches this level. Second, at intermediate scales ($r \sim$ several Mpc), neglecting concentration-dependent clustering will lead to differences in the calculation of the dark matter correlation function due to a combination of the preferential clustering and the convolution factors.

In § 4.1, we derived the normalization relation for the standard mass-dependent halo bias and an analogous relation holds for the concentration-dependent relative halo bias $b_{c_{\rm vir}}$ that we use in this paper. We can write the halo-halo cross correlation factor in terms of a concentration-dependent relative bias of halos with respect to all halos at fixed mass defined so that $\xi_{\rm hh}(r|m_1,c_1,m_2,c_2) = b^2_{c_{\rm vir}}(c_1,c_2,|m_1,m_2)\xi_{\rm hh}(r|m_1,m_2)$. Counting pairs of all halos of fixed mass must give the same result regardless of whether or not we subdivide the halo population by concentration at fixed mass. This requirement gives the general normalization condition

$$\int {\rm d}c_1 \int {\rm d}c_2\, P(c_1|m_1)\, P(c_2|m_2)\, b^2_{c_{\rm vir}}(c_1,c_2|m_1,m_2) = 1\,. \tag{15}$$

Assuming that we can write the bias term as $b^2_{c_{\rm vir}}(c_1,c_2|m_1,m_2) = b_{c_{\rm vir}}(c_1|m_1)b_{c_{\rm vir}}(c_2|m_2)$ as in models of deterministic bias, the normalization condition is then

$$\int {\rm d}cP(c|m)b_{c_{\rm vir}}(c|m) = 1\,. \tag{16}$$

This normalization is consistent with our measurements of the relative concentration-dependent bias, and holds to within a few percent for the fitting formulae of § 3.2.

4.4. *Galaxy Clustering with Concentration-Dependent Bias*

As with the standard halo model, a halo model incorporating concentration-dependent halo clustering can be formally extended to galaxy clustering in a simple manner, though this extension may become cumbersome in practice. As a first attempt, we may assume that the number of galaxies per halo is independent of halo concentration so that as in the previous discussion the substitutions $m \to \langle N_{\rm gal}\rangle_m$ and $m^2 \to \langle N_{\rm gal}(N_{\rm gal}-1)\rangle_m/2$ can be taken in Eq. (14) and Eq. (13). The consequences of the concentration-dependent clustering are then quite similar to the case of the dark matter correlation function, but are less direct because of inherent uncertainties in the link between the matter distribution within halos, $\lambda_m(\vec{x}|c)$, and the distribution of galaxies within halos, $\lambda_m^g(\vec{x}|c)$ (Nagai & Kravtsov 2005; Chen et al. 2005).

Another possibility is that the galaxy HOD does depend on host halo concentration. Indeed, in Figure 5, we confirm that the number of *dark matter* subhalos per host halo at a fixed host halo mass does correlate with the halo concentration, and we have shown that host halo clustering is a function of the number of satellite halos that they contain. A more general and realistic assumption then seems to be to consider galaxy number as a function of both c and m so that the appropriate substitutions into the dark matter correlation function equations [Eq. (13) and Eq. (14)] are $m \to \langle N_{\rm gal}(m,c)\rangle_{m,c}$ and $m^2 \to \langle N_{\rm gal}[N_{\rm gal}-1](m,c)\rangle_{m,c}$. In this case, the

large-scale bias of a particular sample of galaxies can then be written as

$$b_{\rm gal} \simeq \frac{1}{\bar{n}_{\rm g}} \int {\rm d}m\, \frac{{\rm d}n(m)}{{\rm d}m} b_{\rm h}(m) \times \int {\rm d}c\, \langle N_{\rm gal}(m,c)\rangle b_{c_{\rm vir}}(c|m) P(c|m)\,. \quad (17)$$

The bias is weighted over both the concentration distribution and the mass distribution. As such, the model can be compared to the standard halo model using an effective halo occupation, the mean of which is given by

$$\tilde{N}_{\rm gal}(m) = \int {\rm d}c\, P(c|m) b_{c_{\rm vir}}(c|m) \langle N_{\rm gal}(m,c)\rangle\,. \quad (18)$$

Note that because early-forming halos have a lower average satellite number, for luminosity-selected samples it is possible that this could offset the higher bias and result in *galaxy* clustering that does not depend strongly on $c_{\rm vir}$; but this is unlikely to be the case for samples that are more directly connected to formation time.

These arguments can be used and extended to account for any additional dependence of halo clustering on halo properties and corresponding halo occupation distribution, most naturally formation time. Although a larger halo sample will be needed to fully characterize these trends, this first indication of the mass and redshift scaling of the trends of bias with halo properties that we give here should prove useful in order to estimate the size of these effects.

4.5. *General Implications*

Although these results urge caution in using the standard halo model assumptions to calculate galaxy clustering statistics and infer cosmological parameters with high precision, it is not clear that they will have a large effect for galaxy samples that are selected by mass or, as in observational samples, by luminosity. One indication was given by the following test. Zentner et al. (2005) used the standard halo model combined with halo occupation derived from an analytic model for the evolution of halo substructure to predict the two-point correlation function for galaxies, and obtained a result that was consistent with the results of simulations that include all of the effects mentioned here. Still, it may be that small discrepancies would manifest if a larger sample were used to make this comparison. Second, Conroy et al. (2006) have compared estimates of the HOD from galaxy clustering measured in the Sloan Digital Sky Survey (SDSS) (Tinker et al. 2005), where these effects are ignored, with the HOD of subhalos measured in these same simulations, where again these effects are included implicitly, and found that they are in better than $\sim 10\%$ agreement for galaxies brighter than $M_r = -19$.

Other authors have investigated whether the halo occupation is dependent on environment without finding any such trends. For example, Berlind et al. (2003) investigated mean halo occupation as a function of local density in hydrodynamic simulation in a computational box 50 h^{-1}Mpc on a side. These authors measured local density in $4h^{-1}$Mpc spheres around host halos and found no indication of such a trend. Yoo et al. (2005) used the same hydrodynamic simulation as Berlind et al. (2003), to determine whether environment-dependent halo occupation effects could be detected in galaxy correlation statistics. By swapping the galaxy populations between halo populations with similar masses, Yoo et al. (2005) found $5-10\%$ effects on the galaxy-galaxy and galaxy-mass correlations, which were within their statistical uncertainties.

These tests all indicate that the effects for mass- or luminosity-selected samples are at the 10% or lower level. However, we caution that these studies all employed simulations of relatively small volumes. We have performed a comparable test to that shown in Figure 6 using our smaller 60 h^{-1}Mpc box (L60 studied by B01) and found no statistically-significant effect due to the eight-fold smaller computational volume. As such, it is not surprising that these earlier studies were unable to find a conclusive result with their simulation of a yet smaller volume. This indicates that large, high-resolution simulations (suitable for detecting subhalos in $M_{\rm vir} < M_\star$ hosts) will be necessary to determine whether the relative bias measured by halo occupation has the same behavior as might be expected from using concentration in addition to the global relationship between concentration and satellite number as a proxy for halo occupation for all halos in the volume.

These trends have not been readily apparent in observations to date, although a detailed comparison is complicated by the difficulties in selecting a sample that closely corresponds to a mass- and concentration or formation history-selected sample. Recent studies looking at galaxy clustering in the SDSS have not found any trend of clustering with galaxy properties. Skibba et al. (2006) used luminosity-marked correlation functions to test whether the luminosity-dependent clustering is consistent with being a simple consequence of mass-dependent clustering, and found that it is, using $\sim 0.5L*$ galaxies. Abbas & Sheth (2006) showed that the observed environmental dependence of clustering in SDSS could be fully accounted for by correlations between galaxy properties and host mass and by host mass and environment. Another study by Blanton and Berlind (in preparation) shuffled $\gtrsim 0.3L*$ galaxies between groups of the same luminosity and found that the red and blue correlation functions remain unchanged at the $\sim 5\%$ level.

These results offer some confirmation of the theoretical indications that the effects aren't strong in this regime, but are not terribly surprising in light of our results, because for halos with concentrations more than 1σ from the mean, the effect is less than 20% in the range $0.5M_\star < M_{\rm vir} < 10M_\star$. Still, the last study did look at halos well below $M_\star$, and still did not see a strong trend; this may indicate that the correlations between formation time and galaxy observables are not extremely strong. An additional study was performed by Yang, Mo, & van den Bosch (2006), who measured the clustering of a sample of groups selected from the 2dF. They investigated the relative clustering of groups of a given mass as a function of their spectral type, which one might expect to correlate with formation epoch for galaxies at fixed mass, and found that clusters with early-type central galaxies were more clustered than clusters of the same luminosity that had late-type central galaxies. This goes in the sense of our predicted trend for low mass halos but they found that it extended to cluster masses.

A first attempt to connect these trends directly with observable quantities was made by Croton et al. (2006) after this paper was submitted. This paper uses a semi-analytic galaxy formation model to investigate the affect of assembly bias on the total galaxy sample and on samples selected by color, and finds that effects ranging from a few percent to almost a factor of two depending on the selection. At first glance, the predictions of this model do seem to be in mild conflict with the lack of trends seen in the studies above, but this is still unclear. First, shuffling galaxies on observational proxies for mass may lead to much weaker effects than shuffling on the theoretical halo mass. Secondly, the detailed predictions for observed galaxies may dependent sensitively on the galaxy formation model, and it may be that the lack of observed trends indicates that color is less correlated with formation time than in this particular model.

The trends we have presented here are likely to be more important for studies of the clustering of extreme objects that may be thought to form particularly early or late or have particular formation histories. Examples of such populations would be color or star formation selected samples, high-mass clusters selected by occupation number, low surface-brightness galaxies, or dwarf galaxies. We discuss the consequences for such samples further in the next section.

5. SUMMARY AND DISCUSSION

We have investigated the clustering of dark matter halos as a function of several internal halo properties, namely formation time, concentration, and occupation number. We have confirmed that halo clustering is a function of halo formation time and have shown that the effect is scale-dependent. We have also demonstrated that halo clustering is a strong function of halo concentration, and that the strength *and sign* of this trend is a function of mass. Of relevance to studies of galaxy clustering statistics, we also find the clearest indication yet that host halos of fixed mass cluster in a way that is dependent upon the number of subhalos that reside in them. Our primary results can be summarized as follows.

1. Halo clustering is a strong function of formation time for fixed mass halos. This effect strengthens with decreasing halo mass and with decreasing separations, and is an increasingly strong function of mass as halos become less massive than $M_\star$. These results are in broad agreement with the recent results of Sheth & Tormen (2004), Gao et al. (2005) and Harker et al. (2006).

2. We have presented the first definitive measurement showing that the clustering of dark matter halos is a function of halo concentration. This effect is a strong function of halo mass, and can be characterized over a range of mass and redshift as a function of halo mass scaled by the typical collapsing mass, $M_{\rm vir}/M_\star$. Below $M_{\rm vir}/M_\star$, halos of high concentration are more clustered than halos of low concentration and this trend strengthens with decreasing halo mass. For halos more massive than $M_\star$, the trend changes sign, and halos of low concentration become more strongly clustered than their high concentration counterparts.

3. The dependence of halo bias on concentration, mass, and redshift can be parameterized in a simple way: $b(M, c|z) = b(M|z) b_{c_{\rm vir}}(c|M/M_*)$. We provide a fitting function for $b_{c_{\rm vir}}(c|M/M_*)$ that can be used to estimate the importance of these effects in various regimes. In § 4, we demonstrate how this relative bias can be incorporated into a halo model formalism and discuss the effects of concentration- and formation time-dependent bias on estimates of matter and galaxy correlation statistics.

4. We confirm and update earlier results (e.g., Gao et al. 2004; Zentner et al. 2005; van den Bosch et al. 2005; Taylor & Babul 2005) that the occupation number of satellite halos is strongly correlated with both concentration and formation time. We present the first detection of a trend between clustering and halo occupation, showing that at high mass, high-occupation (late-forming) halos are more clustered than their low-N counterparts.

Sheth & Tormen (2004), Gao et al. (2005) and Harker et al. (2006) have emphasized that the trend with formation time indicates that using the halo model to estimate clustering can be problematic. We have quantified this explicitly, by investigating how these trends extend to two variables that are directly relevant to such calculations, namely concentration and halo occupation. There is a weak indication that the trends we see with concentration and halo occupation are slightly stronger than would be predicted simply by their dependence on formation time and the trends with formation time itself, however, this may be do to the larger measurement error in formation time. Larger simulations will be required to determine to higher accuracy whether the strength and nature of the trends with concentration and occupation have features that are not represented by the global correlations between these variables and formation time.

As we emphasized in § 4, the dependence of clustering on concentration implies corrections to halo model calculations of the *dark matter* power spectrum, as well as corrections to halo model calculations of galaxy correlations. These results prescribe caution when using the standard halo model assumptions to calculate galaxy clustering statistics and infer cosmological parameters precisely, but it is not clear that they will have a large effect for samples that are selected by mass or, as in observational samples, by luminosity. This is especially true for galaxies around L_*. For example, Yoo et al. (2005) and Zentner et al. (2005) have both shown that shuffling the host halos of such a sample results in less than about 5% effects in clustering statistics, albeit in relatively small volumes. We expect that the trends we have shown here will have stronger effects on high- or low-mass samples (when compared to $M_\star$) that are selected on some property that is directly connected to either formation time, concentration, or the number of satellite galaxies, and we speculate on a few of these below.

The fact that the clustering of low-mass halos is strongly correlated with formation time may have interesting implications for the so-called "void phenomenon", the tendency of low-mass galaxies to avoid the voids defined by larger galaxies. The currently-favored ΛCDM

model predicts substantial *mass* in voids, so the absence of galaxies in these regions has been suggested to be a potentially serious problem for the prevailing paradigm (Peebles 2001). Whether this is indeed a problem for these models isn't clear; for example, Mathis & White (2002) have investigated the clustering of low luminosity galaxies in LCDM simulations combined with a galaxy formation model and found that all galaxies avoided the voids defined by the brighter galaxies. Benson et al. (2003) found a similar result but emphasized that more data was needed to fully test the issue. Recently, Furlanetto & Piran (2006) compared predictions for void sizes based on the excursion-set formalism with observations from SDSS, and saw indications that the observed voids are somewhat bigger than the model predicts. A related piece of evidence for differential clustering of low-luminosity galaxies has been emphasized by Tully et al. (2002), namely, the difference between the luminosity function in clusters, which rises steeply toward low-luminosity dwarf galaxies, and the luminosity function in voids, which is substantially shallower and appears to be entirely bereft of a dwarf galaxy population.

It has been suggested (Bullock, Kravtsov, & Weinberg 2000) that the discrepancy between the abundance of dwarf satellites observed in the Local Group and the abundance of relevant dwarf mass dark halos ($v_{\rm max} \lesssim 50\ {\rm km\,s^{-1}}$) expected by CDM can be resolved by suppressing galaxy formation in halos that form after the universe is re-ionized. This would bias luminous dwarf galaxies to be associated with late-forming, small dark matter halos. As we have shown, the clustering trend with formation time is quite strong in low-mass halos of this type. If the photoionization significantly affects galaxy formation in dwarf-size galaxy halos, the dwarf galaxies they host would be substantially more clustered than typical halos in the same mass range, and this trend can be enhanced by additional environmental factors (Kravtsov, Gnedin, & Klypin 2004b). If we extrapolate the results of Eq. (6) to $M \sim 10^{-3}\,M_\star$, this implies that the correlation length for the earliest forming quartile should be about a factor of 4 higher than that of the typical halos of that mass. This may provide a natural explanation for the lack of observed dwarf galaxies in voids.

More generally, any astrophysical effect that biases small galaxies to lie in early-forming halos would produce the same effect. There is observational evidence that star formation timescales are quite long ($\sim$ 10Gyr) in small galaxies and relatively short ($\sim$ 1Gyr) in large galaxies (e.g., Searle et al. 1973; Juneau et al. 2005; Willmer et al. 2005, Weiner et al., in preparation). One implication of this is that the total stellar mass accumulated in low-mass halos will be much more sensitive to halo formation time compared to high-mass halos. Such a formation-time bias in the observed properties of low-mass galaxies may not only help to explain the void phenomenon, but will likely be important in attempts to construct conditional luminosity functions that extend to low-luminosity galaxies. The strong clustering of early-forming low-mass halos may also play a part in the observed trend that dim red galaxies are substantially more clustered than their intermediate luminosity counterparts (e.g. Norberg et al. 2002; Hogg et al. 2003), although the basic effect can be explained if the the majority of these galaxies are satellites (Berlind et al. 2005). Of course, any model which forms galaxies using formation histories from a N-body simulation will include the halo clustering effects implicitly, but the extent to which this effects galaxy clustering will depend on the efficiency of galaxy formation in low-mass halos.

There is another interesting implication of our results. Numerous observational signatures indicate that the central densities and concentrations of low surface-brightness (LSB) galaxies are significantly lower than the standard ΛCDM paradigm predicts (e.g., van den Bosch et al. 2000; Debattista & Sellwood 2000; Keeton 2001; van den Bosch & Swaters 2001; Alam et al. 2002; Zentner & Bullock 2002, 2003; McGaugh et al. 2003; van den Bosch et al. 2003a; Kuhlen et al. 2005; Simon et al. 2005). The dependence of clustering on halo concentration may also have implications for the interpretation of the concentrations of LSBs. LSBs have been shown to be notably less clustered than typical galaxies (Mo et al. 1994; Rosenbaum & Bomans 2004, but see also Peebles 2001). This might imply that they reside in a biased population of late-forming, low-concentration halos. If the typical host halos for these galaxies are around $\sim 0.01 M_\star$, and LSB galaxies are about 60% less biased than typical galaxies, this would imply that they have concentration values that are about 1σ below the mean. This would reduce the tension between the predicted and observed concentrations of the halos hosting these galaxies.

Finally, our results also have implications for estimates of the cluster mass function using optically-selected cluster samples. For a richness-selected sample of clusters, or any sample where the primary observable that clusters are selected on is correlated with formation time or halo concentration, mass estimates from "self-calibration", using the clustering of clusters (Majumdar & Mohr 2004; Lima & Hu 2005), may bias results towards higher masses. Note that because these effects correlate with concentration, and probably halo shape and merger history, they could affect SZ- or X-ray-selected samples as well. Similar biases could potentially manifest in weak lensing measurements, and could lead to over-estimates of the mass-to-light ratios in methods that use clustering to constrain the HOD or the conditional luminosity function (e.g. van den Bosch et al. 2003b). They could also impact clustering-based mass estimation for other populations that live in high M/M_* halos, e.g., high-redshift bright galaxies or quasars.

Further work is needed to make quantitative estimates of these effects at high masses, but this trend may be measurable in current optical cluster samples (e.g., from the SDSS, Koester et al., in preparation). The trend between halo occupation and formation time implies that clusters with a given number of galaxies will be a mix of high-concentration, early-forming, high-mass halos and low-concentration, late-forming, low-mass halos. If one can find an observable measure that correlates with formation time (for example, the difference between the luminosity of the first and second brightest cluster galaxies, or the star formation histories of the satellite galaxies), these correlations make specific predictions. At fixed richness or $N_{\rm gal}$, the early-forming sample should be more massive and more concentrated (because concentra-

tion varies much more slowly with halo mass than with formation time: $c \sim m^{0.1}$, Bullock et al. 2001), but less clustered than expected for typical halos of that mass.

Our results indicate that the Universe is somewhat more complicated than our simplest models. However, the complication should be viewed as an opportunity rather than an obstacle, as we can potentially learn a great deal about details of galaxy formation in halos and their evolutionary histories from the trends discussed in this work. The current and upcoming large galaxy surveys (e.g., SDSS, Adelman-McCarthy et al. 2006; DEEP2, Coil et al. 2004; DES, Abbott et al. 2005; LSST, and SNAP, Aldering et al. 2004) should be able to accurately evaluate such effects and test the predicted trends.

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